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Math 130 - Fall 2025 - Common Final Exam

Print name:____

Section number:_______ **Instructor's name:**________________________

Directions:

- This exam has 14 questions. Please check that your exam is complete, but otherwise keep this page closed until the start of the exam is called.
- Fill in your name, and your instructor's name.
- It will be graded out of 200 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- You may use a calculator which cannot connect to the internet. The use of any notes or electronic devices other than a calculator is prohibited.
- A formula sheet has been provided with this exam. You may not refer to any other notes during the exam.
- No earbuds, headphones or hoodies may be worn during the exam.

Good luck!

Question:	1	2	3	4	5	6	7	8
Points:	10	10	12	12	16	10	10	12
Score:								

Question:	9	10	11	12	13	14		Total
Points:	16	16	16	18	20	22		200
Score:								

1. (10 points) Find the equation of the tangent line to the graph of

$$y = \frac{3x^2 - 1}{x + 2}$$

at $x = 1$.

2. (10 points) Solve the equation $8e^{3x-2} = 5e^{x+4}$ using logarithms. Round your final answer to three decimal places.

3. A manufacturing company models the total **production cost** (in thousands of dollars) for producing q units of a custom component by the function

$$C(q) = 2q^3e^{-0.5q} + 4q.$$

(a) (8 points) Find the **marginal cost function** $C'(q)$.

(b) (4 points) Compute the **marginal cost** when $q = 5$ units are produced. Include appropriate units and round your answer to three decimal places.

4. (12 points) Compute the **present value** P of a contract that promises two payments: one of \$4,000 three years from now and one of \$7,000 six years from now. Assume a continuous annual interest rate of 4.2%.

5. A community theater is studying ticket pricing. The table shows the estimated demand $q = D(p)$ (number of tickets sold) at various price points p (in dollars).

p	8.00	8.50	9.00	9.50	10.00	10.50	11.00
q	1200	1100	1000	920	840	780	730

(a) (4 points) Find the **average rate of change** of demand $q = D(p)$ on the interval $[8, 11]$. Explain the real-world meaning in a sentence, including appropriate units.

(b) (8 points) **Estimate the elasticity of demand** at the price $p = 10$. You may use any reasonable differences to approximate $D'(10)$ Interpret your result in a sentence.

(c) (4 points) At $p = 10$, is demand elastic or inelastic? If price increases slightly from \$10, does **revenue** increase or decrease? Explain briefly.

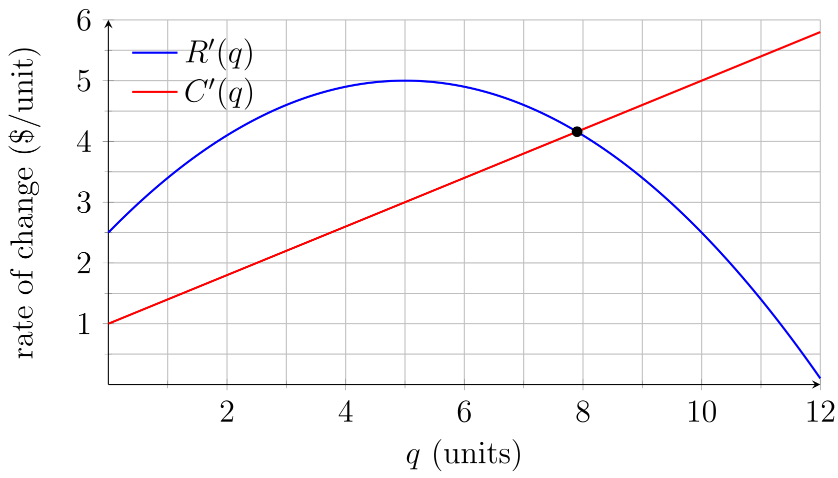
6. (10 points) A recent college graduate wants to begin saving for a future home purchase. They plan to deposit a constant amount S dollars continuously each year into an account that earns interest at a continuous rate of 6.4%. How much must be invested per year so that the account will contain \$150,000 after 20 years?

7. Let $r(t)$ be the rate of change of the price of a share of ABC Inc. in dollars per day after t days of trading.

   (a) (6 points) Write a sentence with units which gives the meaning of $\int_0^6 r(t)\,dt = -7.5$.

   (b) (4 points) If a share of ABC Inc. cost \$25 on day $t = 0$ of trading, use the information above to find its price on day $t = 6$.

8. The curves below represent the *rates of change* of cost and revenue as functions of quantity q: the **marginal cost** $C'(q) = 1 + 0.4q$ and **marginal revenue** $R'(q) = 5 - 0.1(q-5)^2$.



(a) (4 points) For which values of q does revenue increase faster than cost? (Answer in interval form.)

(b) (4 points) Estimate the production level q^* where profit is maximized.

(c) (4 points) Estimate the change in profit if production increases from $q = 3$ to $q = 4$.

9. Consider the function
$$f(x) = x^3 - 6x^2.$$

(a) (4 points) Find all critical points of $f(x)$ Give the critical point and its function value as an (x, y) pair.

(b) (4 points) For each of the critical points found in part (a), determine if it corresponds to a local maximum, local minimum or neither. You may use either the first or second derivative test to justify your answer.

(c) (4 points) Find the absolute (or global) maximum and absolute (or global) minimum of $f(x)$ on the interval $-1 \leq x \leq 7$.

(d) (4 points) Find all value(s) of x for which $f''(x) = 0$ and determine whether or not these value(s) are inflection points.

10. The cost (in thousands of dollars) of producing q tons of a material is

$$C(q) = q^3 - 12q^2 + 60q$$

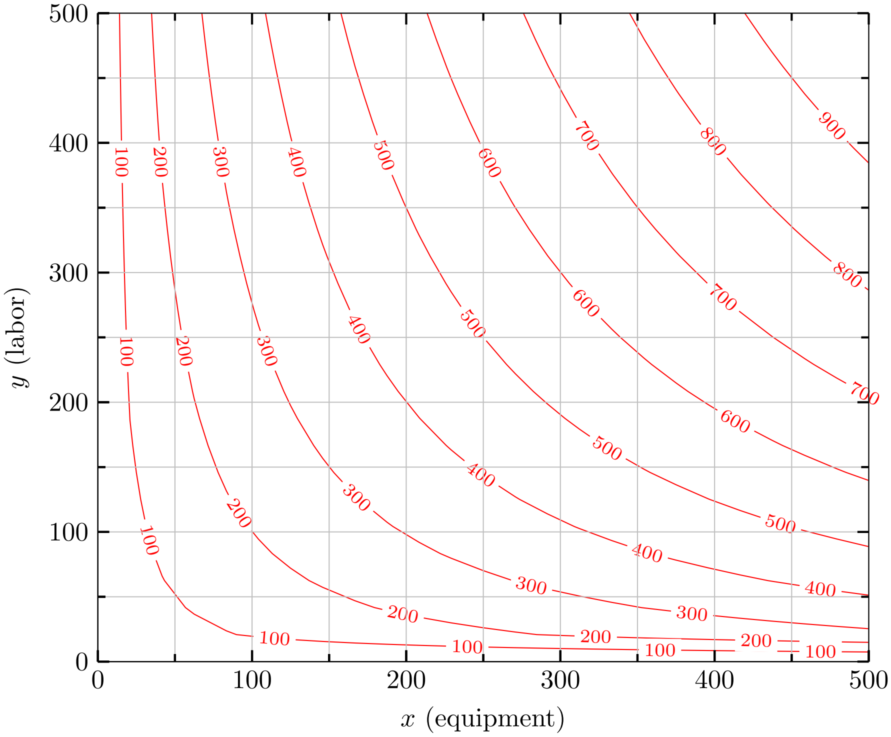
(a) (4 points) Find a formula for the average cost $a(q)$.

(b) (4 points) Find a formula for the marginal cost $MC(q)$.

(c) (4 points) Find all production levels where $a(q) = MC(q)$.

(d) (4 points) Use the values you found in the previous part to determine the production level which minimizes the average cost.

11. A firm produces a single product with two inputs: equipment and labor. The contour plot below shows values of $f(x,y)$, the production level with x dollars in equipment and y dollars in labor.



(a) (6 points) Estimate $f(350,100)$ and $f_y(350,100)$.

(b) (4 points) Explain the meaning of $f(350,100)$ and $f_y(350,100)$ in a sentence with correct units.

(c) (6 points) Mark the **greater** quantity in each pair below.

i. ◯ $f(200,10)$ ◯ $f(200,300)$

ii. ◯ $f_x(200,10)$ ◯ $f_y(200,300)$

iii. ◯ $f_y(200,10)$ ◯ $f_y(200,300)$

12. Let $f(x,y) = (x + xy)^{10}$, and compute the following partial derivatives.

(a) (6 points) $f_x(x,y)$

(b) (6 points) $f_{xx}(x,y)$

(c) (6 points) $f_{xy}(x,y)$

13. A firm manufactures two products with one priced at x dollars and one at y dollars. At prices x and y, the firm's combined revenue for the two products is

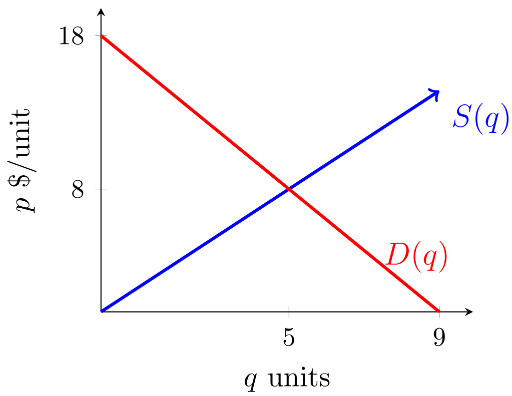
$$R = f(x, y) = -4x^2 - 4xy - 3y^2 + 100x + 130y$$

(a) (8 points) Find the first- and second-order partial derivatives f_x, f_y, f_{xx}, f_{yy} and f_{xy} of the revenue function.

(b) (6 points) Use your partial derivatives to find any critical points for the revenue function. Show your work.

(c) (6 points) Find prices x and y that give the largest possible revenue. Include some computations that show that the revenue is maximized at these prices.

14. The graph below shows supply and demand graphs for a good.



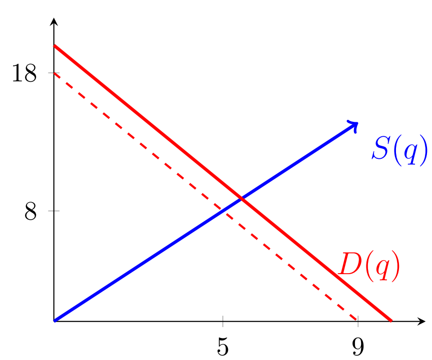
(a) (6 points) Use geometry to find exact values for $\int_0^5 S(q)\,dq$ and $\int_0^5 D(q)\,dq$.

(b) (4 points) What is the equilibrium price and quantity for this market?

$(p^*, q^*) =$____________

(c) (6 points) Compute the value of the consumer surplus.

(d) (6 points) Suppose the demand curve moves from the position shown above to a new position below.



Mark all of the following statements that are correct. Compared the to original demand curve ...

- ◯ $p*$ will increase.
- ◯ $q*$ will increase.
- ◯ the consumer surplus will increase.

Formulas

Derivatives:

- $\dfrac{d}{dx}(cf(x)) = cf'(x)$
- $\dfrac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$
- $\dfrac{d}{dx}(f(x) \cdot g(x)) = f'(x)g(x) + f(x)g'(x)$
- $\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$
- $\dfrac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$
- $\dfrac{d}{dx}(c) = 0$, if c is a constant
- $\dfrac{d}{dx}(mx + b) = m$, where m, b are constants
- $\dfrac{d}{dx}(x) = 1$
- $\dfrac{d}{dx}(x^n) = nx^{n-1}$
- $\dfrac{d}{dx}(a^x) = a^x \cdot \ln(a)$
- $\dfrac{d}{dx}(e^x) = e^x$
- $\dfrac{d}{dx}\left(e^{kx}\right) = k \cdot e^{kx}$, if k is a constant
- $\dfrac{d}{dx}(\ln(x)) = \dfrac{1}{x}$

Other things:

- Quadratic formula: $ax^2 + bx + c = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Future and present value, yearly compounding: $FV = PV \cdot (1 + r)^t$
- Future and present value, continuously compounding: $FV = PV \cdot e^{rt}$
- Elasticity: $E = \left|\dfrac{p}{q} \cdot \dfrac{dq}{dp}\right|$
- Consumer surplus: $\displaystyle\int_0^{q^*} D(q)\, dq - p^*q^*$
- Producer surplus: $\displaystyle p^*q^* - \int_0^{q^*} S(q)\, dq$
- Present value of income stream $S(t)$:

$$PV = \int_0^M S(t)e^{-rt}\, dt$$

- Future value of income stream $S(t)$:

$$FV = e^{rM} \cdot PV = \int_0^M S(t)e^{r(M-t)}\, dt$$

- Second derivative test:
  - Compute discriminant at critical points (a, b):

    $$D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - (f_{xy}(a, b))^2$$

  - $D(a, b) > 0$ and $f_{xx}(a, b) > 0 \Rightarrow$ local min
  - $D(a, b) > 0$ and $f_{xx}(a, b) < 0 \Rightarrow$ local max
  - $D(a, b) < 0$ no local max/min (Saddle Point)
  - $D(a, b) = 0$ inconclusive test